A model for fermion mass generation in Technicolor theory

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Abstract

We consider the $SU(N_{TC})$ Farhi - Susskind Technicolor model, in which $SU(2)$ doublets of technifermions are right-handed while $SU(2)$ singlets of technifermions are left-handed. We add coupling of fermions and technifermions to $SU(N_{TC})$ fundamental massive scalar fields. Due to this coupling the transitions between both types of fermions occur. Therefore the Standard Model fermions acquire masses.

1 Introduction

It is commonly believed that due to the so-called Hierarchy problem the Standard Model (SM) does not work at the energies above 1 Tev[1]. In addition the indications are found that the lattice Weinberg - Salam Model cannot have in principle the ultraviolet cutoff larger than about 1 Tev[2, 3]. That's why it is interesting to consider the models, in which SM Higgs field appears as a composite field. One of the most popular schemes is given within the Technicolor theory (TC), which provides dynamical Electroweak symmetry breaking due to the spontaneous chiral symmetry breakdown and the formation of the condensate for the technifermions[4, 5, 6]. However, Technicolor theory alone cannot provide Standard Model fermions with realistic masses. Usually, in order to make SM fermions massive the so-called Extended Technicolor (ETC) is introduced[7, 8, 9, 10, 11], which provides transition between SM fermions and technifermions. Due to this transition SM fermions are coupled to the condensate of technifermions and become massive. However, this scheme has several difficulties. Dangerous FCNC as well as contributions to Electroweak polarization operators appear. Thus, for example, the realistic masses for SM fermions cannot be generated without appearance of the physical effects excluded by the present measurements. Probably, a possible way

to overcome these difficulties is related to the consideration of the so-called walking technicolor[12, 13, 14, 15] which, however, helps reasonably well for all fermions except for the t quark. Another problem related to Extended Technicolor is that a lot of other Higgs fields are to appear in order to break ETC gauge group down to TC gauge group. The origin of these fields and the pattern of the correspondent breakdown is not considered in sufficient details at the present moment.

In the present paper we suggest an alternative scenario of SM fermions mass formation. Namely, we consider the analogue of $SU(N_{TC})$ Farhi-Susskind Technicolor model[16], in which $SU(2)$ doublets are right-handed while $SU(2)$ singlets are left - handed. With this change made the Technicolor model leads to the same masses of W and Z bosons as the original one. At the same time we are able to introduce an additional coupling of SM fermions and tech-nifermions to $SU(N_{TC})$ massive scalars. The action for these scalars does not contain dangerous ϕ^4 interactions. Therefore, problems specific for the Higgs field of the SM are avoided. We assume the masses of the mentioned scalars are essentially higher than the Technicolor scale. Therefore, integrating out these fields we obtain the low energy effective four - fermion interactions that allow coupling of SM fermions to technifermion condensates. Thus, masses of the SM fermions appear in a complete analogy with ETC models. However, the given approach has several advances. In particular, additional FCNC do not appear at all. It is worth mentioning that in our approach neutrino mass matrix is of Dirac - type. Therefore, some other physics is to be added if it is necessary to obtain Majorana - type neutrino mass matrix.

2   A model for one generation of quarks

Let us consider first the model that contains quarks and techniquarks of one generation. We arrange them in the following $SU(2)$ doublets and singlets.

Quarks:

$$l^{\omega i} = \frac{1-\gamma^5}{2}\left(\begin{array}{c} u^\omega \\ d^\omega \end{array}\right) = \left(\begin{array}{c} u_L^\omega \\ d_L^\omega \end{array}\right) \; ; \; u_R^\omega = \frac{1+\gamma^5}{2}u^\omega \; ; \; d_R^\omega = \frac{1+\gamma^5}{2}d^\omega \qquad (1)$$

Techniquarks:

$$R^{\omega a i} = \frac{1+\gamma^5}{2}\left(\begin{array}{c} U^{\omega a} \\ D^{\omega a} \end{array}\right) = \left(\begin{array}{c} U_R^{\omega a} \\ D_R^{\omega a} \end{array}\right) \; ; \; U_L^{\omega a} = \frac{1-\gamma^5}{2}U^{\omega a} \; ; \; D_L^{\omega a} = \frac{1-\gamma^5}{2}D^{\omega a}$$

$$(2)$$

Here a is $SU(N_{TC})$ index; i is $SU(2)$ index; ω is color $SU(3)$ index. Hypercharge assignment for left - handed (right-handed) technifermions is identical to that of the right - handed (left-handed) SM fermions. Contrary to usual formulation of Technicolor the $SU(2)$ doublets of techniquarks are right-handed while singlets are left - handed. This does not change, however, the pattern of chiral symmetry breaking and the values of masses of W and Z bosons generated. (For general description of how vacuum alignment works in Technicolor theories see, for example, [17].)

We also introduce $SU(N_{TC})$ fundamental scalar H^a. It is assumed that the action for H^a has the form:

$$S_H = \int (DH^+ DH + \frac{M^2}{4} H^2) d^4 x \tag{3}$$

We consider the action for massless quarks and Techniquarks with the additional interaction term

$$S_I = \int \{ (\bar{l}_{\omega i} R^{\omega a i} + \bar{u}_{R,\omega} U_L^{\omega a}) H_a^+ + (\bar{R}_{\omega a i} l^{\omega i} + \bar{U}_{L,\omega a} u_R^\omega) H^a \} d^4 x \tag{4}$$

Technicolor provides chiral symmetry breaking. Therefore, Electroweak symmetry is broken, W and Z bosons become massive and techniquarks are condensed:

$$< \bar{U}_{L,\omega a} R^{\rho a i} > = \quad < \bar{R}_{\rho a i} U_L^{\omega a} > = \frac{1}{6} \delta_1^i \delta_\rho^\omega < \bar{U} U > = -\frac{1}{6} \delta_1^i \delta_\rho^\omega \Lambda_{TC}^3$$

$$< \bar{D}_{L,\omega a} R^{\rho a i} > = \quad < \bar{R}_{\rho a i} D_L^{\omega a} > = \frac{1}{6} \delta_2^i \delta_\rho^\omega < \bar{D} D > = -\frac{1}{6} \delta_2^i \delta_\rho^\omega \Lambda_{TC}^3 \tag{5}$$

Here $\Lambda_{TC} \sim 1$ Tev is at the Technicolor scale.

We suppose $M >> \Lambda_{TC}$ and at low energies integration over H leads to effective four - fermion interaction term:

$$S_I = -\frac{1}{M^2} \int (\bar{l}_i R^{ai} + \bar{u}_R U_L^a)(\bar{R}_{ai} l^i + \bar{U}_{L,a} u_R) d^4 x \tag{6}$$

Using Fierz rearrangement we rewrite this term in the following form:

$$S_I = \frac{1}{M^2} \int \{ -(\bar{l}_i R^{ai})(\bar{R}_{ai} l^i) - (\bar{u}_R U_L^a)(\bar{U}_{L,a} u_R) + \frac{1}{2} [(\bar{l}_{\omega i} u_R^\rho)(\bar{U}_{L,\rho a} R^{\omega a i})$$

$$-\frac{1}{4} (\bar{l}_{\omega i} \gamma^{[\mu} \gamma^{\nu]} u_R^\rho)(\bar{U}_{L,\rho a} \gamma_{[\mu} \gamma_{\nu]} R^{\omega a i}) + (c.c.)] \} d^4 x \tag{7}$$

Here $(c.c.)$ means complex conjugate.

From (7) it follows that the mass term for u - quark appears with the mass

$$m_u = -\frac{1}{6M^2} < \bar{U}_{L,a}R^{a1} >= \frac{\Lambda_{TC}^3}{12M^2} \qquad (8)$$

3 The model with all SM fermions

Now let us turn to the whole Standard Model. We make the following notations:

$$\nu^b = \begin{pmatrix} \nu \\ \nu_\mu \\ \nu_\tau \end{pmatrix} ; u^{\omega b} = \begin{pmatrix} u^1 & u^2 & u^3 \\ c^1 & c^2 & c^3 \\ t^1 & t^2 & t^3 \end{pmatrix}$$

$$e^b = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} ; d^{\omega b} = \begin{pmatrix} d^1 & d^2 & d^3 \\ s^1 & s^2 & s^3 \\ b^1 & b^2 & b^3 \end{pmatrix} \qquad (9)$$

Here index ω runs over $1, 2, 3$. So the model contains the following fields:

Quarks:

$$l^{\omega bi} = \frac{1-\gamma^5}{2}\begin{pmatrix} u^{\omega b} \\ d^{\omega b} \end{pmatrix} = \begin{pmatrix} u_L^{\omega b} \\ d_L^{\omega b} \end{pmatrix} ; u_R^{\omega b} = \frac{1+\gamma^5}{2}u^{\omega b} ; d_R^{\omega b} = \frac{1+\gamma^5}{2}d^{\omega b}$$

$$(10)$$

Leptons:

$$l^{bi} = \frac{1-\gamma^5}{2}\begin{pmatrix} \nu^b \\ e^b \end{pmatrix} = \begin{pmatrix} \nu_L^b \\ e_L^b \end{pmatrix} ; \nu_R^b = \frac{1+\gamma^5}{2}\nu^b ; e_R^b = \frac{1+\gamma^5}{2}e^b \qquad (11)$$

Techniquarks:

$$R^{\omega ai} = \frac{1+\gamma^5}{2}\begin{pmatrix} U^{\omega a} \\ D^{\omega a} \end{pmatrix} = \begin{pmatrix} U_R^{\omega a} \\ D_R^{\omega ai} \end{pmatrix} ; U_L^{\omega a} = \frac{1-\gamma^5}{2}U^{\omega a} ; D_L^{\omega a} = \frac{1-\gamma^5}{2}D^{\omega a}$$

$$(12)$$

Technileptons:

$$R^{ai} = \frac{1+\gamma^5}{2}\begin{pmatrix} N^a \\ E^a \end{pmatrix} = \begin{pmatrix} N_R^a \\ E_R^a \end{pmatrix} ; N_L^a = \frac{1-\gamma^5}{2}N^a ; E_L^a = \frac{1-\gamma^5}{2}E^a$$

$$(13)$$

Here we have the only generation of technifermions. Index a belongs to $SU(N_{TC})$. Hypercharge assignment for left - handed (right-handed) technifermions is identical to that of the right - handed (left-handed) SM fermions.

Let us consider the following interaction term:

$$S_I = \int \{\delta_c^b \bar{l}_{\omega bi} R^{\omega ai} + \eta_c^b \bar{u}_{R,\omega,b} U_L^{\omega a} + \lambda_c^b \bar{d}_{R,\omega,b} D_L^{\omega a}\} H_a^c d^4 x$$

$$+ \int \{\delta_c^b \bar{l}_{bi} R^{ai} + \zeta_c^b \bar{\nu}_{R,b} N_L^a + \gamma_c^b \bar{e}_{R,b} E_L^a\} H_a^c d^4 x$$

$$+ (c.c.) \tag{14}$$

Here η, λ, ζ, and γ are hermitian matrices of coupling constants. We have introduced 3 $SU(N_{TC})$ scalar fields H^c with the common action

$$S_H = \int ([DH_a^c]^+ DH_a^c + \frac{M^2}{4}[H_a^c]^+ H_a^c) d^4 x \tag{15}$$

Integrating out scalar fields we obtain low energy effective four - fermion interactions:

$$S_I = -\frac{1}{M^2} \int \{\delta_c^b \bar{l}_{\omega bi} R^{\omega ai} + \eta_c^b \bar{u}_{R,\omega,b} U_L^{\omega a} + \lambda_c^b \bar{d}_{R,\omega,b} D_L^{\omega a}$$

$$+ \delta_c^b \bar{l}_{bi} R^{ai} + \zeta_c^b \bar{\nu}_{R,b} N_L^a + \gamma_c^b \bar{e}_{R,b} E_L^a\}^+$$

$$\{\delta_c^b \bar{l}_{\omega bi} R^{\omega ai} + \eta_c^b \bar{u}_{R,\omega,b} U_L^{\omega a} + \lambda_c^b \bar{d}_{R,\omega,b} D_L^{\omega a}$$

$$+ \delta_c^b \bar{l}_{bi} R^{ai} + \zeta_c^b \bar{\nu}_{R,b} N_L^a + \gamma_c^b \bar{e}_{R,b} E_L^a\} d^4 x \tag{16}$$

After Fierz rearrangement in a similar way to the previous section we obtain mass matrices for the SM fermions:

$$m_u = -\frac{1}{12M^2} < \bar{U}U > \eta$$

$$m_d = -\frac{1}{12M^2} < \bar{D}D > \lambda$$

$$m_\nu = -\frac{1}{4M^2} < \bar{N}N > \zeta$$

$$m_e = -\frac{1}{4M^2} < \bar{E}E > \gamma \tag{17}$$

We can diagonalize these matrices via an appropriate unitary transformation of leptons and quarks of different generations. During this diagonalization mixing in the charged currents appears as usual.

4   Conclusions

In this paper we considered the alternative (to ETC) scheme of SM mass generation for the Farhi-Susskind technicolor model. Additional $SU(N_{TC})$

scalar fields are introduced with masses well above Technicolor scale. Due to coupling of these scalars to SM fermions and technifermions the transition between both kinds of fermions appears. As a result SM fermions become massive. Fermion mixing also appears in this scheme in a natural way. The action for the mentioned scalar fields does not contain ϕ^4 interaction. Therefore, the given model does not suffer from the problems specific for the SM Higgs sector.

Probably, the main advance of the given approach is that all additional terms in effective action (16) contain transition between SM fermions and technifermions. The additional four - fermion terms that contain only SM particles do not appear. That's why a lot of difficulties due to appearance of such terms in ETC models are avoided.

Nevertheless, several problems related to the suggested scheme remain unsolved. In particular, the consideration of possible additional effects due to the terms appeared in (16) are in order. Also it is important to consider carefully the question about the renormalizability of the given model and quantum anomalies. All these issues, however, are considered to be out of the scope of the present paper[1].

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